1.8 Interpreting Graphs of Functions

Learning Targets:
1. Interpret intercepts and symmetry of graphs
2. Interpret positive, negative, increasing, decreasing, extrema, and end behavior.
1. Is the relation a function?

A. yes
B. no
2. Is the relation a function?

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>-8</td>
</tr>
<tr>
<td>12</td>
<td>-6</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-4</td>
<td>2</td>
</tr>
<tr>
<td>-10</td>
<td>5</td>
</tr>
</tbody>
</table>

- A. yes
- B. no
4 Is the relation $y = 6$ a function?

A. yes

B. no
5. If $f(x) = 3x + 7$, find $f(10)$.

A. 20

B. 23

C. 32

D. 37
6. If \( g(x) = -2x - 2 \), find \( g(-2x) \).

A. \( g(-2x) = 4x - 2 \)

B. \( g(-2x) = 4x^2 + 4 \)

C. \( g(-2x) = 4x^2 \)

D. \( g(-2x) = 4x^2 - 2 \)
7. What is the range shown in this function table?

<table>
<thead>
<tr>
<th>$x$</th>
<th>$4x + 5$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2</td>
<td>$4(-2) + 5$</td>
<td>−3</td>
</tr>
<tr>
<td>0</td>
<td>$4(0) + 5$</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>$4(1) + 5$</td>
<td>9</td>
</tr>
</tbody>
</table>

A. {−3, 5, 9}
B. {−8, 0, 4}
C. {3, 5, 6}
D. {−2, 0, 1}
Mathematical Practices

1 Make sense of problems and persevere in solving them.

Content Standards

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
Then

You identified functions and found function values.

Now

• Interpret intercepts, and symmetry of graphs of functions.

• Interpret positive, negative, increasing, and decreasing behavior, extrema, and end behavior of graphs of functions.
New Vocabulary

- intercept - points where a graph intersects an axis.
- x-intercept - where the graph crosses the x axis
- y-intercept - where the graph crosses the y axis
- line symmetry - each half of the graph matches on either side
- positive - where the graph lies above the x axis
- negative - where the graph lies below the x axis

- increasing - where the graph goes up (left to right)
- decreasing - where the graph goes down (left to right)
- extrema - high or low points
- relative minimum - low point
- relative maximum - high point
- end behavior - describes the values at the ends of the graph. (look at the arrows)
**Example 1**

**Interpret Intercepts**

**Salad**  The graph shows the cost \( y \) of a salad weighing \( x \) ounces at a salad bar. Identify the function as **linear** or **nonlinear**. Then estimate and interpret the intercepts of the graph of the function.
Example 1

SALES  The graph shows the expected revenue $y$ as a function of the price per unit $x$. Identify the function as linear or nonlinear. Then estimate and interpret the intercepts of the graph of the function.

A. nonlinear; (0, 100): Zero revenue at a price of $100; (62, 0): $62 expected revenue at a cost of $0.
B. nonlinear; (100, 0): Zero revenue at a price of $100; (0, 62): $62 expected revenue at a cost of $0.
C. nonlinear; (100, 0): revenue at a price of $0; (0, 62): No expected revenue at a price of $62 per unit.
D. nonlinear; (0, 100): revenue at a price of $0; (62, 0): No expected revenue at a price of $62 per unit.
Example 2

SALES  The graph shows the expected revenue $y$ as a function of the price per unit $x$. Describe and interpret any symmetry.

A. The right half of the graph is the mirror image of the left half in approximately the line $x = 30$. The expected revenue at a price of $n$ more or $n$ less than 30 dollars per unit will be the same.

B. The right half of the graph is the mirror image of the left half in approximately the line $x = 3$. The expected revenue at a price of $n$ more or $n$ less than 3 dollars per unit will be the same.

C. There is no symmetry.

D. The right half of the graph is the mirror image of the left half in approximately the line $y = 1000$. The expected price at a revenue of $n$ more or $n$ less than 1000 dollars per unit will be the same.
**Key Concepts** Positive, Negative, Increasing, Decreasing, Extrema, and End Behavior

A function is **positive** where its graph lies above the x-axis, and **negative** where its graph lies below the x-axis.

A function is **increasing** where the graph goes up and **decreasing** where the graph goes down when viewed from left to right.

The points shown are the locations of relatively high or low function values called **extrema**. Point A is a **relative minimum**, since no other nearby points have a lesser y-coordinate. Point B is a **relative maximum**, since no other nearby points have a greater y-coordinate.

**End behavior** describes the values of a function at the positive and negative extremes in its domain.

As you move left, the graph goes up. As x decreases, y increases.

As you move right, the graph goes down. As x increases, y decreases.
Example 3  Interpret Extrema and End Behavior

DEER  The graph shows the population $y$ of deer $x$ years after the animals are introduced on an island. Estimate and interpret where the function is positive, negative, increasing, and decreasing, the $x$-coordinates of any relative extrema, and the end behavior of the graph.
Example 3  Interpret Extrema and End Behavior

Answer:
Positive:  between $x = 0$ and about $x = 5.5$
Negative:  for about $x > 5.5$

This means that the population died out about 5.5 years after the deer were introduced to the island.
Increasing:  for between about $x = 1$ and $x = 4$
Decreasing:  for $x < 1$ and $x > 4$

This means that the deer population decreased for the first year, increased for the next three years, then decreased after year 4.

Relative Maximum:  at about $x = 4$
Relative Minimum:  at about $x = 1$

The extrema of the graph indicate that the population experienced a relative low at year 1 and a relative high at year 4. The population then decreased to 0 after year 4.
Example 3

**SPEED** The graph shows the speed of a car after $x$ minutes. Estimate and interpret where the function is positive, negative, increasing, and decreasing, the $x$-coordinates of any relative extrema, and the end behavior of the graph.

End Behavior

As $x$ inc.
As $x$ dec
As $x$ inc
As $x$ dec
Example 3

A. \(0 < t < 7.2\), positive: \(t > 7.2\) minutes;
   decreasing: \(t < 1\), \(4 < t < 6\), increasing: \(1 < t < 4\),
   \(t > 6\), max: \(t = 4\), min: \(t = 1\) and \(t = 6\)

B. \(0 < t < 7.2\), negative: \(t > 7.2\) minutes;
   increasing: \(t < 1\), \(4 < t < 6\), decreasing: \(1 < t < 4\),
   \(t > 6\), max: \(t = 4\), min: \(t = 1\) and \(t = 6\)

C. \(0 < t < 7.2\), positive: \(t > 7.2\) minutes;
   increasing: \(t < 30\), \(90 < t < 120\), decreasing: \(30 < t < 90\), \(t > 120\),
   max: \(t = 1\), min: \(t = 4\)

D. \(0 < t < 7.2\), negative: \(t > 7.2\) minutes;
   increasing: \(t < 1\), \(4 < t < 6\), decreasing: \(1 < t < 4\),
   \(t > 6\), max: \(t = 1\) and \(t = 6\), min: \(t = 4\)
Post Assessment:

p. 61-62 (1-4)

Assignment: p. 62-63 (5-7, 13-15)